PH4442 Advanced Particle Physics 2025/26
Lecture Week 8

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- The electroweak Standard Model
- Decay rate of the Z boson
- Gauge boson self-couplings

Road map to electroweak unification

V−A theory of weak interactions agrees with data, but:

- not renormalisable (infinite predictions from higher orders),
- cross sections exceed unitarity limit at high energy.

Intermediate Vector Boson ~saves unitarity but not obvious how to build into renormalisable theory.

Yang-Mills gauge theories are renormalisable (‘t Hooft 1971), and gauge bosons automatically give IVB.

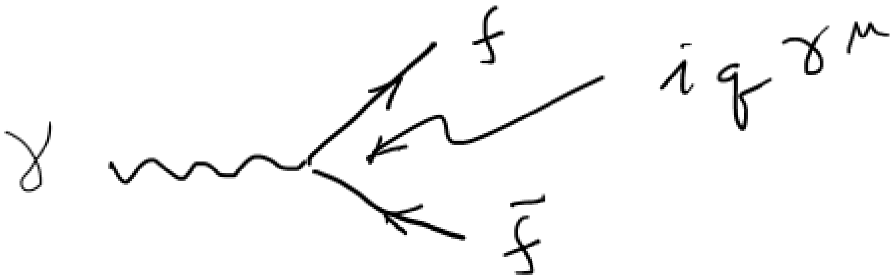
Naively, masses for gauge bosons breaks gauge invariance (see next week). So for now, regard them (and fermions) as massless. Restore later with Higgs mechanism.

We need: parity violation, observed transitions between fermion types, CKM mixing,...

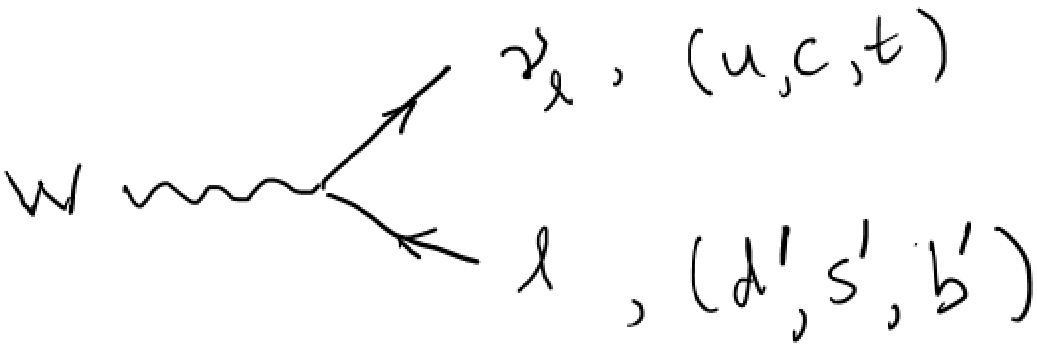
What we get from gauge theory based on SU(2) x U(1): a unified renormalisable theory of weak and electromagnetic interactions; neutral currents (Z) emerge as a prediction.

Photon and W couplings to fermions

fermion-photon: same as QED



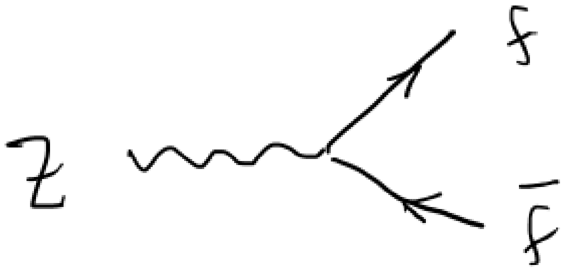
fermion-W:



$$-i\frac{g}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)$$

Or $\quad -i\frac{gV_{ij}}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)$ when down-type is mass eigenstate (d,s,b).

Fermion-Z couplings



$$-i\frac{g}{2\cos\theta_{\rm W}}\gamma^\mu(c_{V,f} - c_{A,f}\gamma^5)$$

vector, axial-vector couplings

or with left- and right chiral couplings:

$$c_{L,f} = c_{V,f} - c_{A,f}$$

$$c_{R,f} = c_{V,f} + c_{A,f}\,.$$

$$\begin{aligned}
c_{V,f} &= T_{3,f} - 2Q_f \sin^2\theta_{\rm W}\ , \\
c_{A,f} &= T_{3,f}\ , \\
c_{L,f} &= T_{3,f} - Q_f \sin^2\theta_{\rm W}\ , \\
c_{R,f} &= -Q_f \sin^2\theta_{\rm W}\ .
\end{aligned}$$

Summary of fermion-Z couplings

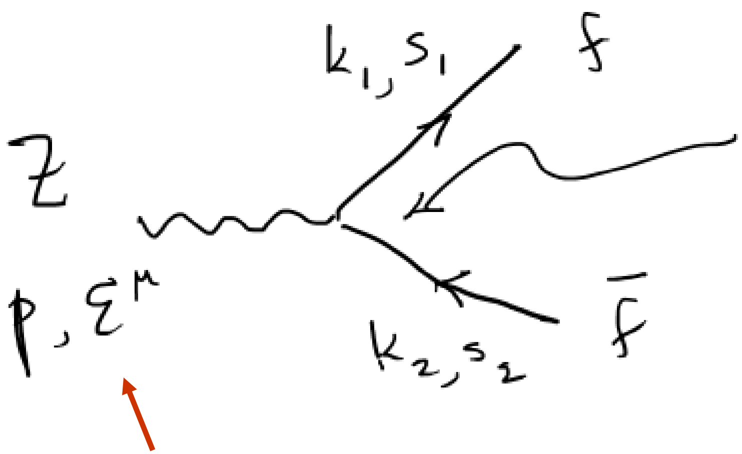
Table 10.1: Z boson couplings to fermions.

Fermion	T_3	Q_f	$c_{V,f}$	$c_{A,f}$	$c_{L,f}$	$c_{R,f}$
ν_e, ν_μ, ν_τ	$+\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
$e^-, \mu^- \tau^-$	$-\frac{1}{2}$	-1	$-\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$	$-\frac{1}{2} + \sin^2\theta_W$	$\sin^2\theta_W$
u, c, t	$+\frac{1}{2}$	$+\frac{2}{3}$	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$	$\frac{1}{2} - \frac{2}{3}\sin^2\theta_W$	$-\frac{2}{3}\sin^2\theta_W$
d, s, b	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$	$-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W$

From experiment: $\sin^2\theta_{\mathrm{W}} = 0.223$.

Decay rate of Z boson

Cheat: use couplings just derived assuming massless particles, apply to Z boson with M_Z = 91.2 GeV, also use $M_W = M_Z \cos\theta_W$ (from Higgs mech., next week).



$$-i\frac{g}{2\cos\theta_W}\gamma^\mu(c_{V,f} - c_{A,f}\gamma^5)$$

polarisation vector

$$\begin{aligned}\mathcal{M} &= \overline{u}(k_1, s_1)\left(\frac{-ig}{2\cos\theta_W}\gamma^\mu(c_V - c_A\gamma^5)\right)v(k_2, s_2)\varepsilon_\mu \\ &= \frac{-ig}{2\cos\theta_W}\overline{u}\not{\varepsilon}(c_V - c_A\gamma^5)v\end{aligned}$$

Z decay rate (2)

Square amplitude and sum over final-state spins with Casimir trick:

$$\sum_{s_1,s_2} |\mathcal{M}|^2 = \sum_{s_1,s_2} \frac{g^2}{4\cos^2\theta_W} \left[\bar{u}\gamma^\mu(c_V - c_A\gamma^5)v\right] \left[\bar{u}\gamma^\nu(c_V - c_A\gamma^5)v\right]^* \varepsilon_\mu \varepsilon_\nu^*$$

$$= \frac{g^2}{4\cos^2\theta_W} \mathrm{Tr}\left[\gamma^\mu(c_V - c_A\gamma^5)\not{k}_2 \overline{\gamma^\nu(c_V - c_A\gamma^5)}\not{k}_1\right] \varepsilon_\mu \varepsilon_\nu^* \,.$$

$$= \sum_{s_1,s_2} \frac{g^2}{4\cos^2\theta_W} \left[\bar{u}\gamma^\mu(c_V - c_A\gamma^5)v\right] \left[\bar{u}\gamma^\nu(c_V - c_A\gamma^5)v\right]^* \varepsilon_\mu \varepsilon_\nu^*$$

$= \gamma^\nu(c_V - c_A\gamma^5)$

$$= \frac{g^2}{4\cos^2\theta_W} \mathrm{Tr}\left[\gamma^\mu(c_V - c_A\gamma^5)\not{k}_2 \overline{\gamma^\nu(c_V - c_A\gamma^5)}\not{k}_1\right] \varepsilon_\mu \varepsilon_\nu^*$$

Move second factor of $(c_V - c_A\gamma^5)$ to the left past $\not{k}_2\gamma^\nu$

$\rightarrow$ two sign changes to the γ^5 term that cancel

$$(c_V - c_A\gamma^5)^2 = c_V^2 - 2c_V c_A\gamma^5 + c_A^2(\gamma^5)^2 = c_V^2 + c_A^2 - 2c_V c_A\gamma^5$$

Z decay rate (3)

Spin-summed squared amplitude becomes:

$$\sum_{s_1,s_2} |\mathcal{M}|^2 = \frac{g^2}{4\cos^2\theta_W} \mathrm{Tr}\left[(c_V^2 + c_A^2 + 2c_V c_A \gamma^5) \not{\varepsilon} \not{k}_2 \not{\varepsilon}^* \not{k}_1\right]$$

Evaluate traces:

$$\sum_{s_1,s_2} |\mathcal{M}|^2 = \frac{g^2}{\cos^2\theta_W}\Big[(c_V^2 + c_A^2)\left((\varepsilon\cdot k_2)(\varepsilon^*\cdot k_1) + (\varepsilon\cdot k_1)(\varepsilon^*\cdot k_2) - (\varepsilon\cdot\varepsilon^*)(k_1\cdot k_2)\right)$$
$$- \quad 2c_V c_A i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu k_2^\nu \varepsilon^{\rho *} k_1^\sigma\Big]$$

Suppose Z polarised along z-axis: $\varepsilon^\mu = (0,0,0,1), \quad \varepsilon^* = \varepsilon$

$\varepsilon^\mu k_2^\nu \varepsilon^{\rho *} k_1^\sigma$ is symmetric under interchange of μ and ρ.

$\epsilon_{\mu\nu\rho\sigma}$ is antisymmetric under $\mu \leftrightarrow \rho$,

$\rightarrow \quad \varepsilon^\mu k_2^\nu \varepsilon^{\rho *} k_1^\sigma \epsilon_{\mu\nu\rho\sigma} = 0 \qquad \rightarrow$ term with $c_V c_A$ is zero

Z decay rate (4)

For the chosen polarisation we also have:

$$\varepsilon \cdot k_1 = \varepsilon^* \cdot k_1, \; \varepsilon \cdot k_2 = \varepsilon^* \cdot k_2, \; \varepsilon \cdot \varepsilon^* = \varepsilon_\mu \varepsilon^\mu = -1$$

$$\rightarrow \quad \sum_{s_1, s_2} |\mathcal{M}|^2 = \frac{g^2(c_V^2 + c_A^2)}{\cos^2\theta_W} \left[2(\varepsilon \cdot k_1)(\varepsilon \cdot k_2) + k_1 \cdot k_2 \right]$$

Go to c.m. of Z boson, approximate $m_f \ll M_Z$ (OK for all fermions except top quark)

$$\rightarrow \quad |\boldsymbol{k}_1| = |\boldsymbol{k}_2| \approx \frac{M_Z}{2}$$

$$\begin{aligned} k_1 &= \frac{M_Z}{2}(1, \sin\theta, 0, \cos\theta) \,, \\ k_2 &= \frac{M_Z}{2}(1, -\sin\theta, 0, -\cos\theta) \,, \end{aligned}$$

θ = polar angle of outgoing fermion

Z decay rate (5)

In c.m., spin-summed squared amplitude becomes

$$\sum_{s_1,s_2} |\mathcal{M}|^2 = \frac{g^2(c_V^2 + c_A^2)M_Z^2}{2\cos^2\theta_W} \sin^2\theta \, .$$

Use formula for two-body decay rate in c.m.:

$$p^* = |\boldsymbol{k}_1| \approx M_Z/2$$

$$d\Gamma = \frac{1}{32\pi^2 M^2} \sum_{s_1,s_2} |\mathcal{M}|^2 p^* \, d\Omega$$

Independent of azimuthal angle $d\Omega = 2\pi d\cos\theta$

$$\rightarrow \qquad \frac{d\Gamma}{d\cos\theta} = \frac{g^2(c_V^2 + c_A^2)M_Z}{64\pi\cos^2\theta_W} \sin^2\theta$$

Z decay rate (6)

Integrate over $\cos\theta$ to find total decay rate $\Gamma(Z \to f\overline{f})$

Use, e.g., $x = \cos\theta$ so $\sin^2\theta = 1 - x^2$, $\int_{-1}^{1} (1 - x^2)\, dx = 4/3$

$$\rightarrow \qquad \Gamma(Z \to f\overline{f}) = \frac{g^2 (c_V^2 + c_A^2) M_Z}{48\pi \cos^2\theta_W}$$

Because direction of z axis arbitrary, this gives the total rate to fermion pairs for any polarisation of the Z.

Can re-express using $$G_{\rm F} = \frac{\sqrt{2}}{8} \frac{g^2}{M_W^2} = \frac{\sqrt{2}}{8} \frac{g^2}{M_Z^2 \cos^2\theta_W^2}$$

$$\Gamma(Z \to f\overline{f}) = \frac{G_{\rm F} M_Z^3 (c_V^2 + c_A^2)}{6\pi\sqrt{2}}$$

Z decay rate (7)

Using measured values of the parameters:

$$\begin{aligned} M_Z &= 91.2\,\mathrm{GeV} \\ G_\mathrm{F} &= 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2} \\ \sin^2\theta_W &= 0.233 \end{aligned}$$

gives (for first family):

$$\begin{aligned} \Gamma(Z \to \nu_e \overline{\nu}_e) &= 166\,\mathrm{MeV} \\ \Gamma(Z \to e^+ e^-) &= 83.9\,\mathrm{MeV} \\ \Gamma(Z \to u\overline{u}) &= 96.6\,\mathrm{MeV} \\ \Gamma(Z \to d\overline{d}) &= 124\,\mathrm{MeV} \end{aligned}$$

Z decay rate (8)

For total Z decay rate, sum over fermion types except top, include N_c = 3 colours for quarks (cf. Ch. 12 on QCD), suppose N_ν = 3 neutrino families:

$$\Gamma_Z = N_\nu \Gamma(Z \to \nu_e \overline{\nu}_e) + 3\Gamma(Z \to e^+ e^-) + 2N_C \Gamma(Z \to u\overline{u}) + 3N_C \Gamma(Z \to d\overline{d}) = 2.44\,\mathrm{GeV}$$

Predicted value with higher-order EW, QCD, mass corrections:

$$\Gamma_Z = 2.4940 \pm 0.0009\,\mathrm{GeV},$$

Measured value (LEP):

$$\Gamma_Z = 2.4955 \pm 0.0023\,\mathrm{GeV}$$

Impressive confirmation of N_ν = 3 neutrino families.

Gauge boson self couplings (brief overview)

Because $SU(2)_L$ non-abelian, gauge bosons interact with each other. (contrast to QED: photon has no charge).

Recall for QED (Maxwell eqs.): $\partial_\mu F^{\mu\nu} = j^\nu$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

Corresponding equations for $\boldsymbol{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ are

$$\partial_\nu F^{\mu\nu,a} - g\epsilon_{abc} W_\nu^b F^{\mu\nu,c} = J^{\mu,a} ,$$

source current from fermion and Higgs fields

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\epsilon_{abc} W_\mu^b W_\nu^c ,$$

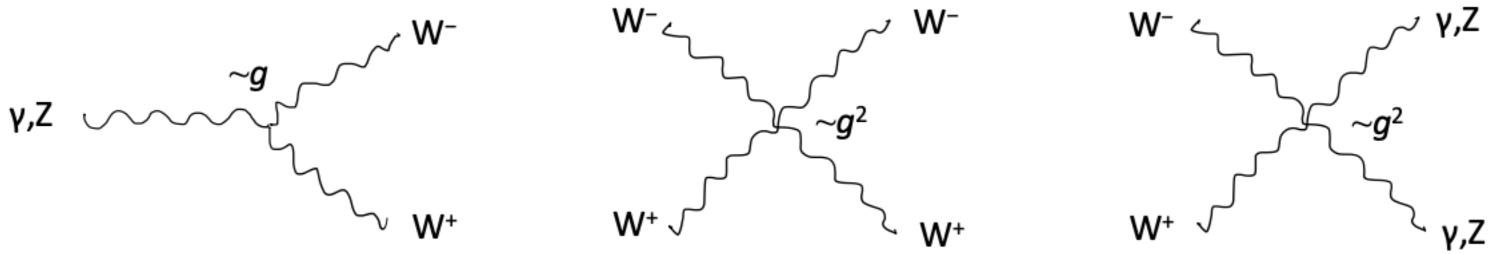
Even if no fermion or Higgs fields, the term $g\epsilon_{abc} W_\nu^b F^{\mu\nu,c}$ plays role of a source current.

Amplitudes from interaction Hamiltonian $H_{\text{int}} \sim g\epsilon_{abc} W_\mu^a W_\nu^b F^{\mu\nu,c}$

have terms with 3 and 4 factors of the fields $\boldsymbol{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$

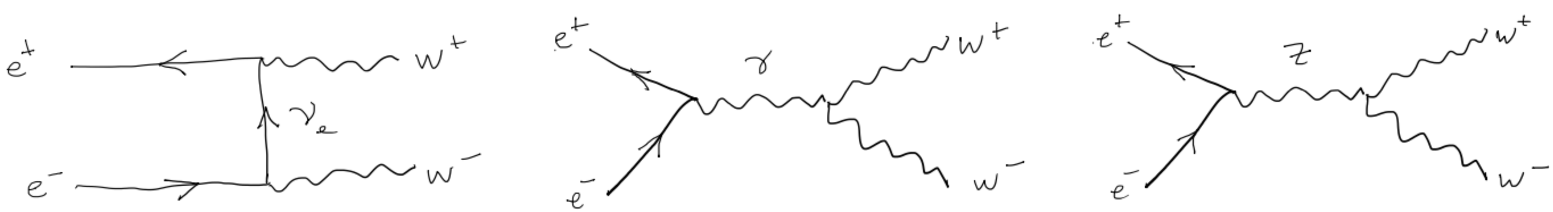
Gauge boson self couplings

These lead to triple and quartic gauge boson self couplings:



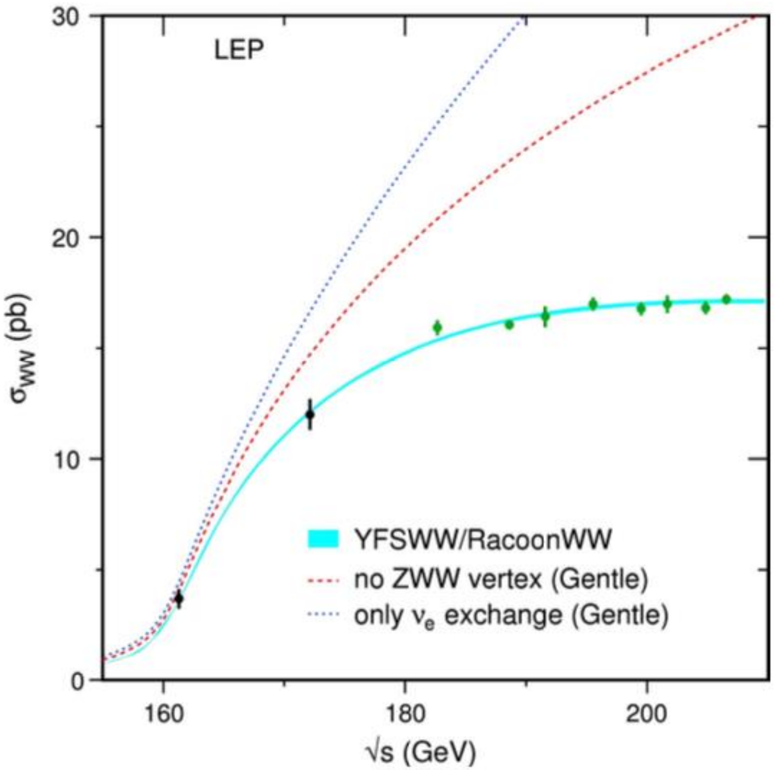
Note still no coupling with only Z, photon.

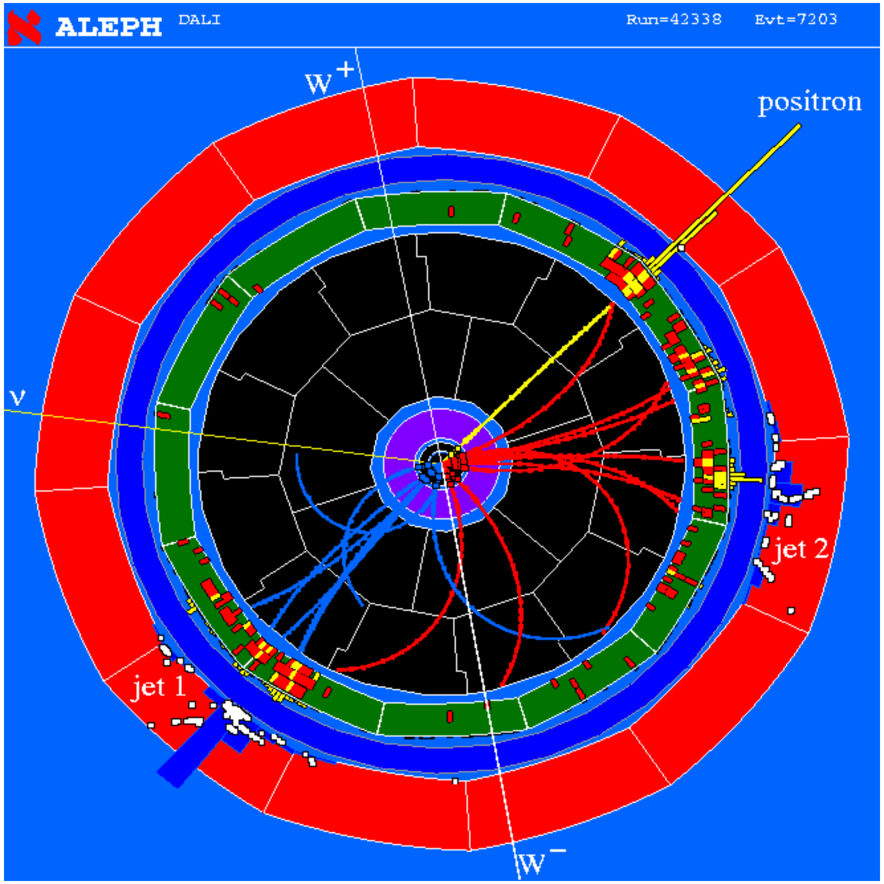
Important experimental test: $e^+e^- \to W^+W^-$



$e^+e^- \longrightarrow W^+W^-$ at LEP

Measurements of cross section $\sigma(e^+e^- \to W^+W^-)$ vs. E_{cm} at LEP:





The neutrino-exchange diagram interferes destructively with triple gauge-boson vertex ones; resulting prediction in excellent agreement with data.