PH4442 Advanced Particle Physics 2025/26
Lecture Week 9

Glen Cowan
Physics Department
Royal Holloway, University of London
g.cowan@rhul.ac.uk
www.pp.rhul.ac.uk/~cowan

- Brief intro to Quantum Field Theory
- The Higgs Mechanism
- Properties of the Higgs boson

Plan for this week

Last week got electroweak model based on $SU(2)_L$ x U(1) local gauge symmetry, but only worked for massless particles.

But we know SM particles have masses!

This week: show how to included masses while retaining gauge symmetry with the Higgs mechanism.

First, introduce some ideas of Quantum Field Theory.

Then look at a "toy" model with a U(1) gauge symmetry to see how the Higgs mechanism works.

Then apply to the full Standard Model:

→ masses for W, Z and also fermions in a renormalisable theory,

plus a new scalar boson (the Higgs),

plus other testable predictions (predict M_W).

Lagrangian formalism

Consider system with *N* generalised coordinates and velocities

$$\boldsymbol{q} = (q_1, \ldots, q_N)$$

$$\dot{\boldsymbol{q}} = (\dot{q}_1, \ldots, \dot{q}_N)$$

Evolves from t_1 to t_2 such that the action *S* is minimised,

$$S = \int_{t_1}^{t_2} L(\boldsymbol{q}, \dot{\boldsymbol{q}})\, dt$$

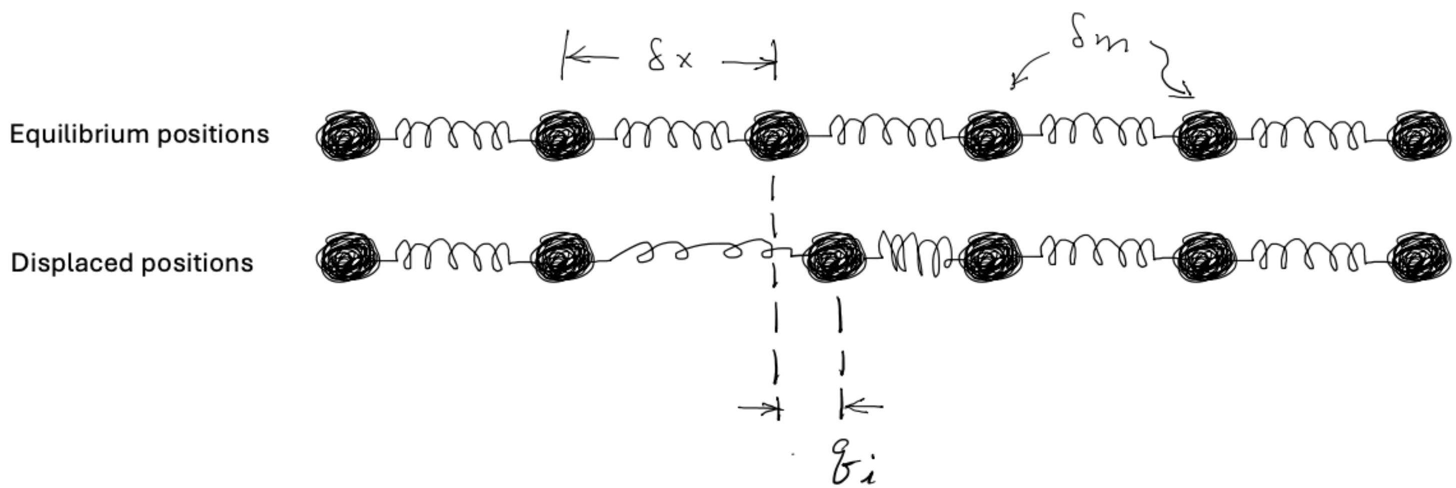
Lagrangian $L = T - V$

This leads to the Euler-Lagrange equations:

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0\,, \qquad i = 1, \ldots, N$$

Lagrangian for a string of masses

Consider a system of a large number of masses δm separated by distances δx connected by springs of spring constant k.



$$L = \sum_{i=1}^{N} \left(\tfrac{1}{2} \delta m \, \dot{q}_i^2 - \tfrac{1}{2} k (q_{i+1} - q_i)^2 \right)$$

$$= \sum_{i=1}^{N} \frac{\delta x}{2} \left[\left(\frac{\delta m}{\delta x} \right) \dot{q}_i^2 - k \, \delta x \left(\frac{q_{i+1} - q_i}{\delta x} \right)^2 \right] \quad = \sum_{i=1}^{N} \delta x \, L_i$$

From system of masses to a field

Consider the limit where the number of masses N becomes very large and δm and δx go to zero, such that $\delta m/\delta x \to \rho$ and $k\delta x \to \tau$ approach constants.

The index i measures the x coordinate of the ith mass and the displacement $q_i \equiv \varphi(x)$ becomes a function of the continuous variable x, i.e., it is a *field*.

The Lagrangian becomes

Lagrangian density

$$L \to \int dx \, \frac{1}{2} \left[\rho \dot{\phi}^2(x) - \tau \left(\frac{\partial \phi}{\partial x} \right)^2 \right] \equiv \int dx \, \mathcal{L}(x)$$

The Euler –Lagrange equations become $$\frac{\partial \mathcal{L}}{\partial \phi(x)} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)} \right) = 0$$

which for the string of masses give a wave equation:

$$\tau \left(\frac{\partial^2 \phi}{\partial x^2} \right) - \rho \left(\frac{\partial^2 \phi}{\partial t^2} \right) = 0 \qquad v = \sqrt{\tau/\rho}.$$

From fields in 1-D to 4-D space-time

Suppose now system in 3-D space, Lagrangian is

$$S = \int_{t_1}^{t_2} dt \int d^3x \, \mathcal{L}(\phi, \partial_\mu \phi)$$

To be consistent with relativity, Lagrangian density (usually just called the Lagrangian) is allowed to depend on the field and on its derivatives with respect to space and time.

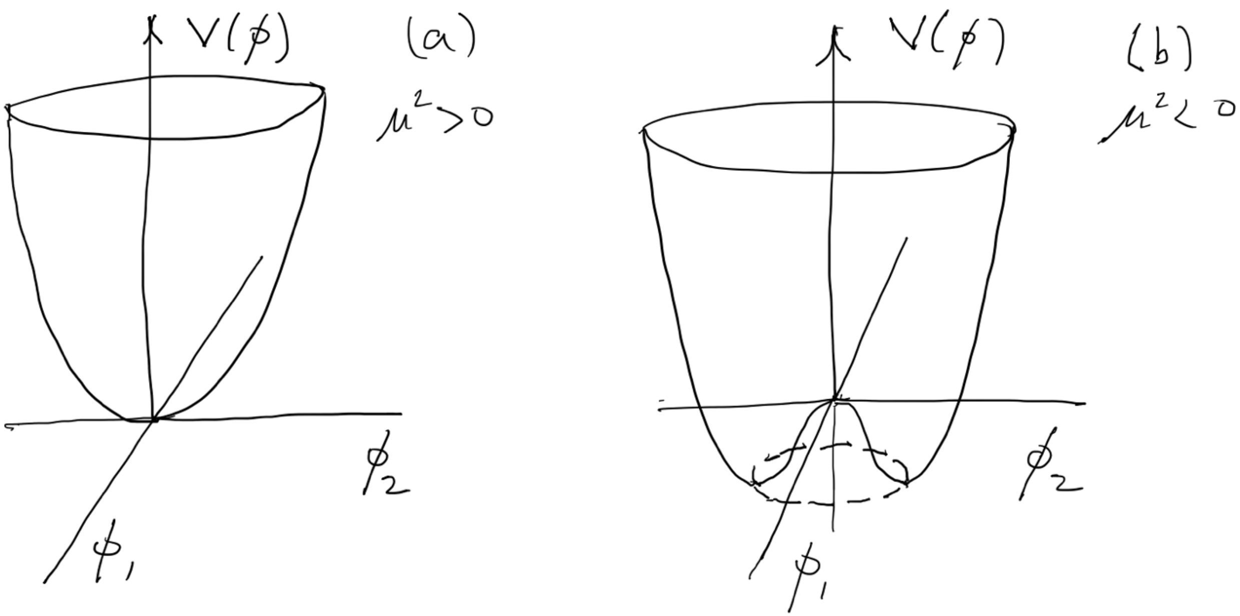
The Euler-Lagrange equations become

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0$$

The Higgs Potential (single complex scalar)

$$\phi = (\phi_1 + i\phi_2)/\sqrt{2}$$

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$



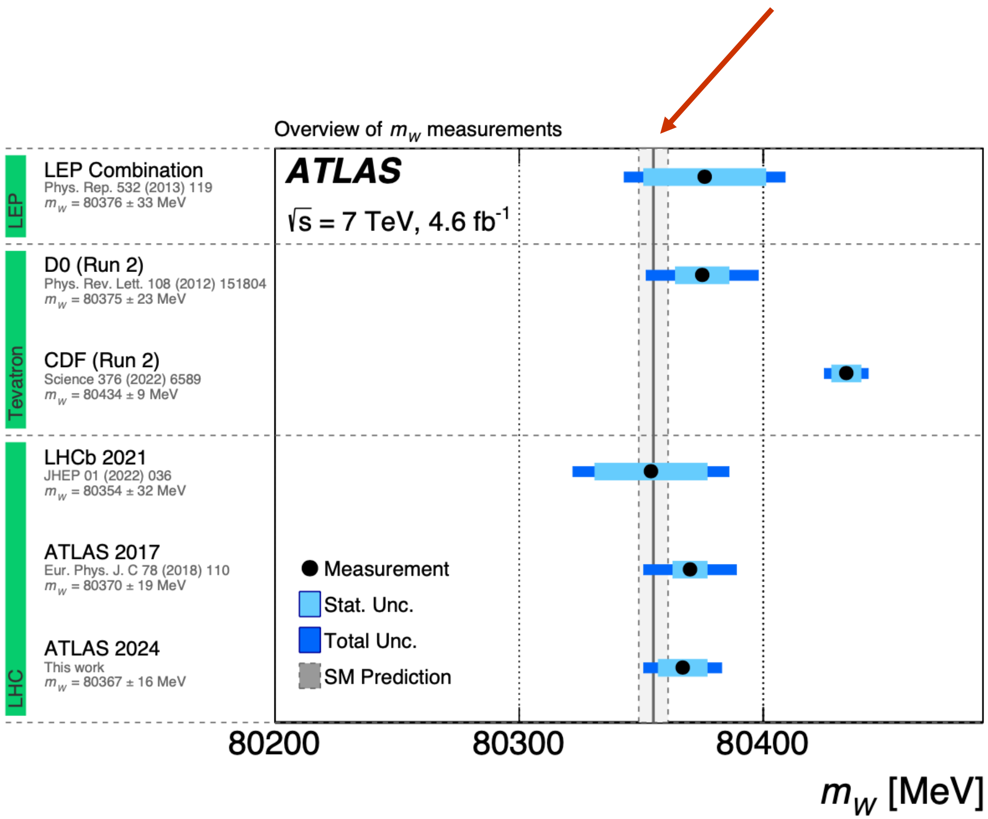
For $\mu^2 < 0$, V minimum at

$$\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} \equiv v^2$$

Prediction for M_W

Higgs mechanism leads to $$\frac{M_W}{M_Z} = \cos\theta_W$$

from which we can predict: $$M_W = 80.3545 \pm 0.0057 \text{ GeV}$$



Excellent* agreement with experiment

*2022 measurement by CDF out by 7.2σ

The mass of the Higgs boson

In the Lagrangian multiply out the terms $-\frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4$

This gives a quadratic term $-\frac{1}{2}\left(\mu^2 + 3\lambda v^2\right) h^2 = -\frac{1}{2} m_{\rm H}^2 h^2$

from which we identify the Higgs mass $m_{\rm H} = \sqrt{2\lambda} v = \sqrt{-2\mu^2}$.

Before the discovery of the Higgs in 2012, $v = 2M_W/g = 246\,\mathrm{GeV}$

was known from G_F, but λ and therefore also m_H were not known.

After Higgs discovery find m_H = 125 GeV $\rightarrow$ λ = 0.129.

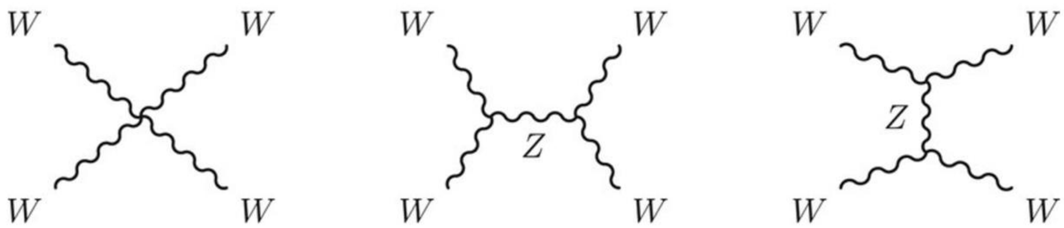
But even before discovery of Higgs, theory was widely accepted because:

- Renormalisable
- Prediction of M_W confirmed
- Solves unitarity bound of σ(WW→WW)

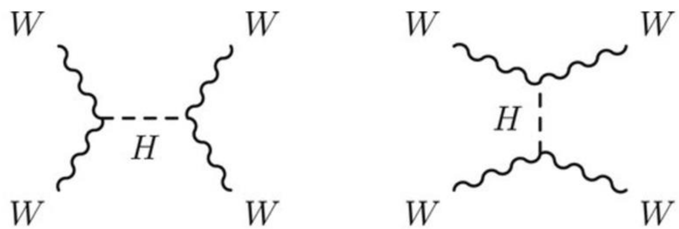
$W_L W_L \to W_L W_L$ scattering at high energy

Without Higgs, $\sigma(W_L W_L \to W_L W_L) \sim E_{cm}^2$.

Violates unitarity limit at $E_{cm} \sim 1$ TeV



With Higgs, extra diagrams interfere destructively,



cancelling E_{cm}^2 dependence and keeping cross section from exceeding unitarity bound.

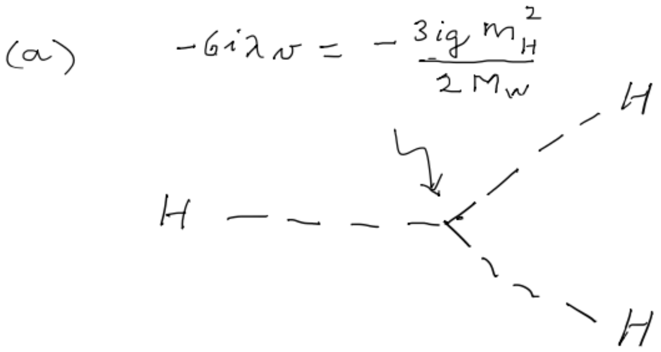
Higgs self coupling

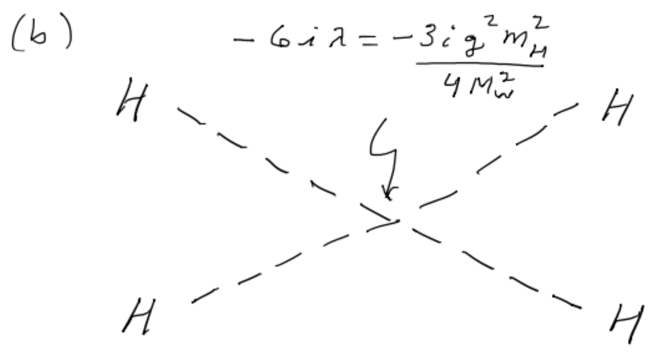
Multiplying out $-(\lambda/4)(v+h)^4$ gives $-\lambda v h^3 - \frac{\lambda}{4}h^4$

These correspond to triple and quartic H couplings, vertex factors:

Triple, *N*=3 identical particles: $$-6i\lambda v = -\frac{3im_{\rm H}^2}{v} = -\frac{3igm_{\rm H}^2}{2M_W}$$

Quartic, *N*=4 identical particles: $$-6i\lambda = -\frac{3im_{\rm H}^2}{v^2} = -\frac{3ig^2m_{\rm H}^2}{4M_W^2}$$



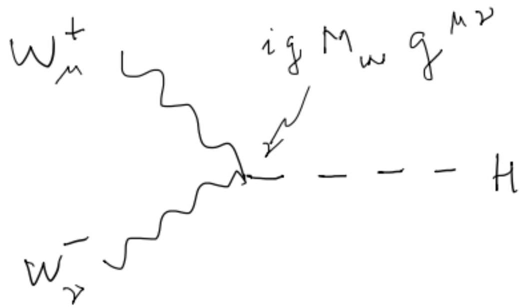


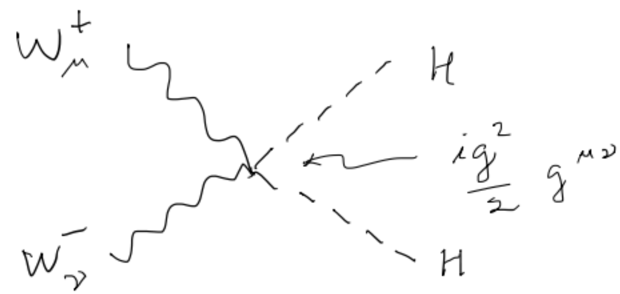
Higgs couplings to gauge bosons

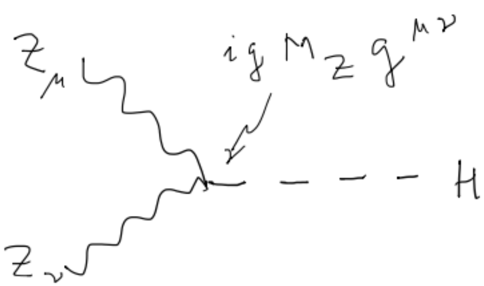
Multiplying out terms with $(v+h)^2$ and $(v+h)^4$ in Lagrangian and expressing W_1, W_2, W_3, B in terms of W^+, W^-, A, Z gives interactions between Higgs and gauge bosons, e.g.,

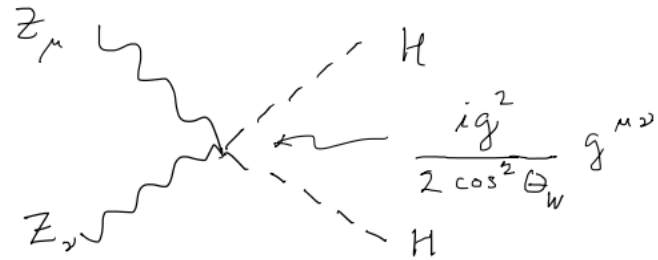
$$\tfrac{1}{2} g^2 v W_\mu^{(-)} W^{(+)\mu} h + \tfrac{1}{4} g^2 W_\mu^{(-)} W^{(+)\mu} h^2$$

and analogous for Z, but no direct coupling to photon.









Coupling of the Higgs to fermions

So far got masses for W,Z. But we also need fermion masses.
We cannot just add fermion mass terms to the Lagrangian.

Consider $\psi = \psi_L + \psi_R$ where

$$\psi_L = P_L\psi = \frac{1}{2}(1 - \gamma^5)\psi$$

$$\psi_R = P_R\psi = \frac{1}{2}(1 + \gamma^5)\psi$$

Recall $\gamma^0\gamma^5 = -\gamma^5\gamma^0$ so $\gamma^0 P_L = P_R\gamma^0$ and $\gamma^0 P_R = P_L\gamma^0$

Also $(\gamma^5)^\dagger = \gamma^5$ so $P_L = P_L^\dagger$ and $P_R = P_R^\dagger$

Therefore $\overline{\psi}_L = (P_L\psi)^\dagger\gamma^0 = \psi^\dagger\gamma^0 P_R = \overline{\psi}P_R$ and $\overline{\psi}_R = \overline{\psi}P_L$

For mass term $-m\overline{\psi}\psi$ with $\psi_L + \psi_R$, $\overline{\psi}_L\psi_L = \overline{\psi}P_R P_L\psi = 0$, $\overline{\psi}_R\psi_R = 0$

The only terms that survive are $-m\overline{\psi}\psi = -m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$

But SU(2)L transforms ψ_L and ψ_R differently, so this term breaks the gauge symmetry.

Lepton masses

Consider transformation of scalar $\phi' = U_T \phi \,, \quad U_T = \exp[ig\boldsymbol{\alpha}(x) \cdot \boldsymbol{T}]$

and for left-chiral lepton doublet $L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L$

$$\begin{aligned} L' &= U_T L \,, \\ \overline{L}' &= (U_T L)^\dagger \gamma^0 = L^\dagger \gamma^0 U_T^\dagger = \overline{L} U_T^\dagger \end{aligned}$$

Right-chiral leptons $R = \psi_{\ell,R}$ have $T_3 = 0$ (do not change under $SU(2)_L$.

Under $U(1)_Y$ they transform as $R' = U_Y R$, $U_Y = e^{iY\theta}$, $\theta \equiv g'\beta(x)$

Hypercharge is $Y = 2(Q - T_3)$ so $Y_\phi = 1, Y_L = -1$, $Y_R = -2$, so

$$\begin{aligned} \overline{L} = L^\dagger \gamma^0 \quad &\rightarrow \quad \overline{L}' = (e^{-i\theta} L)^\dagger \gamma^0 = \overline{L} e^{i\theta} \\ \phi \quad &\rightarrow \quad \phi' = e^{i\theta} \phi \\ R \quad &\rightarrow \quad R' = e^{-2i\theta} R \,, \end{aligned}$$

So the combination $(\overline{L}\phi)R$ is invariant under $U(1)_Y$.

Yukawa Lagrangian

R does not change under SU(2)L, so $(\overline{L}\phi)R$ is invariant under $SU(2)_L \times U(1)_Y$. So we can add to the Lagrangian

$$\mathcal{L}_\ell \;=\; -y_\ell \left[\overline{L}\phi R + (\overline{L}\phi R)^\dagger\right]$$

(y_l = Yukawa coupling, include Hermitian conjugate to ensure *L* real.)

After transform to unitary gauge, $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

Using this in the Lagrangian above, the terms involving neutrinos disappear since they are in the upper row of *L* ($T_3 = ½$). Substituting the terms gives

$$\mathcal{L}_\ell = -\frac{y_\ell}{\sqrt{2}} v(\overline{\ell}_L \ell_R + \overline{\ell}_R \ell_L) - \frac{y_\ell}{\sqrt{2}} h(\overline{\ell}_L \ell_R + \overline{\ell}_R \ell_L)$$

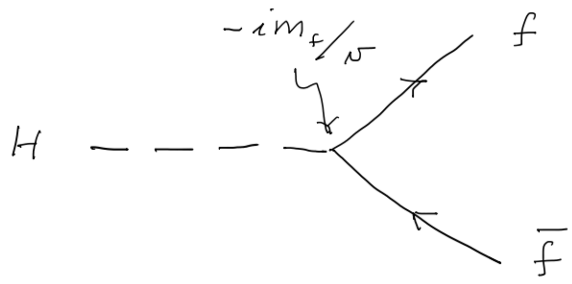
Yukawa coupling

We identify the first term as a mass term $-m_\ell \bar{\psi}\psi$ with $m_\ell = \frac{y_\ell v}{\sqrt{2}}$

Assign the Yukawa coupling y_l so that it gives the experimentally observed masses. No predicted relation between masses, need an adjustable parameter in the model for each one.

Second term in Yukawa Lagrangian is an interaction between the Higgs boson and a lepton pair with vertex factor

$$-iy_\ell/\sqrt{2} = -im_\ell/v$$



Less obvious for up-type quarks but same recipe holds.