Useful Formulae

$$\hbar = 6.58 \times 10^{-25}\ \text{GeV sec} = 1$$
$$\hbar c = 0.197\ \text{GeV F} = 1$$
$$(1\ \text{GeV})^{-2} = 0.389\ \text{mb}$$
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

$$x^\mu = (t, \mathbf{x}), \qquad p^\mu = (E, \mathbf{p}) = i\left(\frac{\partial}{\partial t}, -\nabla\right) = i\partial^\mu$$

$$p \cdot x = Et - \mathbf{p}\cdot\mathbf{x}, \qquad p^2 = p^\mu p_\mu = E^2 - \mathbf{p}^2 = m^2$$

$$(\Box^2 + m^2)\phi = 0, \qquad (i\gamma^\mu\partial_\mu - m)\psi = 0.$$

In an electromagnetic field, $i\partial^\mu \to i\partial^\mu + eA^\mu$ (charge $-e$)

$$j^\mu = -ie(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*), \qquad j^\mu = -e\bar{\psi}\gamma^\mu\psi$$

γ-Matrices

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}, \qquad \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0.$$

$$\gamma^{0\dagger} = \gamma^0, \qquad \gamma^0\gamma^0 = I; \qquad \gamma^{k\dagger} = -\gamma^k, \qquad \gamma^k\gamma^k = -I, \qquad k = 1,2,3.$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad \gamma^\mu\gamma^5 + \gamma^5\gamma^\mu = 0, \qquad \gamma^{5\dagger} = \gamma^5.$$

(Trace theorems on pages 123 and 261)

Standard representation:

$$\gamma^0 \equiv \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \boldsymbol{\gamma} \equiv \beta\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spinors

$$(\not{p} - m)u = 0 \qquad \bar{u} \equiv u^\dagger\gamma^0$$
$$\bar{u}(\not{p} - m) = 0 \qquad \not{p} \equiv \gamma_\mu p^\mu$$

$$u^{(r)\dagger}u^{(s)} = 2E\delta_{rs}, \qquad \bar{u}^{(r)}u^{(s)} = 2m\delta_{rs}, \qquad \sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = \not{p} + m = 2m\Lambda_+$$

$$\tfrac{1}{2}(1-\gamma^5)u \equiv u_L, \qquad \tfrac{1}{2}(1+\gamma^5)u \equiv u_R.$$

If $m = 0$ or $E \gg m$, then u_L has helicity $\lambda = -\frac{1}{2}$, u_R has $\lambda = +\frac{1}{2}$.

Kinematics

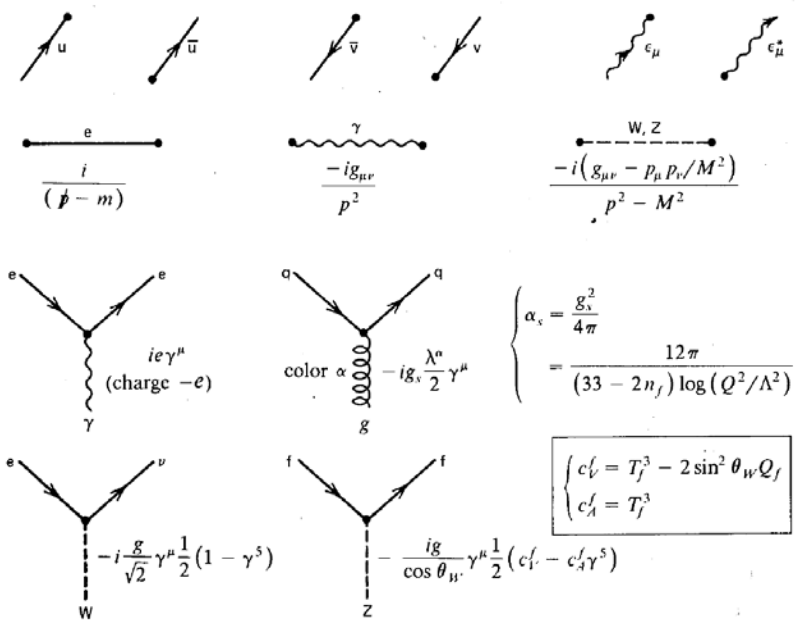
Lorentz invariant phase space ($P \to p_1 + \cdots p_n$)

$$dQ = (2\pi)^4\delta^4(P - p_1 - \cdots p_n)\prod_{i=1}^{n}\frac{d^3p_i}{(2\pi)^3 2E_i}$$

Scattering: $$\left.\frac{d\sigma}{d\Omega}\right|_{\text{cm}} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathfrak{M}|^2$$

Decay: $$d\Gamma(A \to 1 + \cdots n) = \frac{|\mathfrak{M}|^2}{2m_A}dQ.$$

Feynman Rules for $-i\mathfrak{M}$



$$\alpha_s = \frac{g_s^2}{4\pi} = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2)}$$

$$\begin{cases} c_V^f = T_f^3 - 2\sin^2\theta_W Q_f \\ c_A^f = T_f^3 \end{cases}$$

f	Q_f	$(T_f^3)_L$	$(T_f^3)_R$
u, c, t	$+\frac{2}{3}$	$\frac{1}{2}$	0
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	0
ν_e, ν_μ, ν_τ	0	$\frac{1}{2}$	—
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	0

$$\sin^2\theta_W \simeq 0.23,\ g\sin\theta_W = e,\ G = \frac{\sqrt{2}\,g^2}{8M_W^2} \simeq 1.17 \times 10^{-5}\ \text{GeV}^{-2}$$

Halzen & Martin