PH4442 Advanced Particle Physics
Problem sheet 5
Due Monday, 15 December 2025

1 [15 marks] The equation of motion for a field $\varphi(x)$ is obtained from the Lagrangian density $\mathcal{L}(\varphi, \partial_\mu \varphi)$ from the Euler-Lagrange equation,

$$\frac{\partial \mathcal{L}}{\partial \varphi} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \ .$$

(a) [**4 marks**] The Lagrangian density $\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m)\psi$ describes the Dirac fields ψ and $\overline{\psi}$. By considering the Euler-Lagrange equations for $\overline{\psi}$, show that ψ follows the Dirac equation.

(b) [**6 marks**] Apply the Euler-Lagrange equation to the field ψ and show that this gives $i\partial_\mu \overline{\psi}\gamma^\mu + m\overline{\psi} = 0$, and show that this is (up to a sign) the adjoint of the Dirac equation.

(c) [**5 marks**] By forming a combination of $\overline{\psi}$ multiplied on the left with the Dirac equation and the adjoint equation times ψ on the right, show that one finds an equation of the form $\partial_\mu j^\mu = 0$ and find j^μ.

2 [15 marks] Consider a real scalar field ϕ with Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \ , \tag{1}$$

where the potential is

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4 \ . \tag{2}$$

(a) [**1 mark**] Why would we need to assume $\lambda > 0$?

(b) [**2 marks**] Find the value of ϕ that minimises the potential V for $\mu^2 > 0$. For $\mu^2 > 0$, what is the mass of the corresponding field excitation (the "ϕ particle")?

(c) [**3 marks**] Suppose we consider the scattering reaction $\phi\phi \to \phi\phi$. Sketch the leading order Feynman diagram, and from inspection of $\mathcal{L}$ identify the vertex factor.

(d) [**2 marks**] Write down the invariant amplitude for $\phi\phi \to \phi\phi$.

(e) [**3 marks**] Find the differential cross section $d\sigma/d\cos\theta$ in the c.m. frame for $\phi\phi \to \phi\phi$. Sketch $d\sigma/d\cos\theta$ as a function of $\cos\theta$.

(f) [**3 marks**] Suppose the coupling parameter is $\lambda = 0.1$. Find the total cross section and evaluate in fb for a centre-of-mass energy of $E_{\text{cm}} = 100\,\text{TeV}$.

(g) [**1 mark**] Suppose the Large Phi Collider (LPC) delivers an integrated luminosity of $100\,\text{fb}^{-1}$ at a centre-of-mass energy of $E_{\text{cm}} = 100\,\text{TeV}$. What is the expected number of $\phi\phi$ scattering events?

3 [12 marks] Consider again the same Lagrangian as in Ex. 2, but now with $\mu^2 < 0$.

(a) [4] Sketch the potential versus ϕ and indicate the two minima at $\pm v$. Find v as a function of μ^2 and λ.

(b) [**8**] Expand the field about its positive minimum v:

$$\phi(x) = v + \eta(x) \ . \tag{3}$$

Write down the Lagrangian in terms of the new field η. Show it corresponds to a real scalar field and find its mass as a function of μ^2 and λ.

4 [**8 marks**] Consider the quark scattering reaction $ud \rightarrow ud$.

(a) [**2 marks**] Draw the lowest-order Feynman diagram of this reaction indicating all particles and couplings.

(b) [**4 marks**] Labelling the four-momenta of the incoming and outgoing u quarks with p_1 and p_3, and the incoming and outgoing d quarks with p_2 and p_4, respectively, write down the invariant amplitude before any averaging over spin indices.

(c) [**2 marks**] Explain which terms would be different in this amplitude for the case of $e^-\mu^- \rightarrow e^-\mu^-$ scattering.