PH2900 (Astronomy) Problem Sheet 5
Autumn term, 2010
Due date to be announced.

PART MARKS

1. (a) Consider a spherical gas cloud with temperature T, radius R and uniform density ρ consisting of non-interacting particles of mean mass μm_{H}, i.e., μ is the mean molecular weight and m_{H} is the mass of a hydrogen atom. Show that the maximum radius of the cloud it can have before collapsing is given by the Jeans radius,

$$R_{\mathrm{J}} = C \left(\frac{kT}{G \rho \mu m_{\mathrm{H}}} \right)^{1/2} ,$$

where C is a constant of order unity. (Hint: use the virial theorem to relate the kinetic and potential energies of the cloud. You do not need to keep track of numerical factors of order unity.) **[6]**

(b) Suppose a spherical gas cloud of non-interacting particles with uniform density ρ and radius R collapses under its own gravity. Show that the time for collapse is given by the *free-fall time*,

$$t_{\mathrm{ff}} = \left(\frac{3\pi}{32 G \rho} \right)^{1/2} .$$

Use the fact that Kepler's third law gives the period $T = 2\pi a^{3/2} / \sqrt{GM}$ for a particle orbiting with semi-major axis a around a mass M. **[4]**

(c) Describe why in the collapse of the solar nebula the matter winds up following roughly circular orbits in a flattened disc. **[4]**

(d) Explain qualitatively why the objects in the asteroid belt do not collect into a single planet.

Describe briefly the Kirkwood gaps.

Explain qualitatively why the Jovian planets have ring systems.

What is particular about the location of the large gap in Saturn's rings called the Cassini Division? **[6]**

G. Cowan
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