PH2900 (Astronomy)
Problem sheet 3
To be turned in by 5:00 p.m., Wednesday 14 November 2007

1(a): A CCD image of a star field is analysed to determine the brightness of a star. Let N_s be the number of photoelectrons found in a signal region around the star, N_b is the number found in a background region of equal area where there are no stars, and take $N_* = N_\mathrm{s} - N_\mathrm{b}$ as an estimate of the number of photoelectrons from the star itself. Show using error propagation that the relative statistical uncertainty in N_* is

$$\frac{\sigma_{N_*}}{N_*} = \sqrt{\frac{N_* + 2N_b}{N_*^2}} \; .$$

1(b): Suppose the star is very dim, so that we may approximate $N_* \ll N_\mathrm{b}$. Furthermore suppose that owing to the construction of a nearby motorway, the sky brightness changes from 19 to 16.5 magnitudes per square arcsec. By what factor would the relative statistical uncertainty σ_{N_*}/N_* change assuming the same exposure time as before?

1(c): By what factor would you need to increase the exposure time to achieve the same σ_{N_*}/N_* as obtained before construction of the motorway?

2: Suppose we take the source function for the sun's photosphere to be of the form

$$S(\tau) = a + b\tau \; , \tag{1}$$

where a and b are constants and we will ignore the dependence of S and τ on wavelength. By substituting (1) into equation (6.25) from the lecture notes, show that one obtains for the intensity emerging from the sun, $I(0, \mu)$,

$$I(0, \mu) = a + b\mu \; , \tag{2}$$

or equivalently,

$$I(0, \mu) = S(\tau = \mu) \; , \tag{3}$$

which is called the Eddington-Barbier relation. That is, if you look at the centre of the sun's disc at $\mu = \cos\theta = 1$, you see the intensity given by the source function at an optical depth of 1.

3(a): The star Betelgeuse has a temperature of $T = 3400$ K and Rigel has $T = 10,100$ K. Find the peak wavelengths in nm of the light emitted by both stars by treating them as blackbodies. What colours do these wavelengths correspond to?

3(b): On a clear (!?) night go look at Rigel and Betelgeuse and verify your answer to part (a). (Betelgeuse is the upper left 'shoulder' of Orion; Rigel is the lower-right foot.)

G. Cowan
31 October 2007