Introduction to Statistics
CERN Summer Student Lectures
Problem sheet #1

We can discuss the exercises during the discussion session on Tuesday; otherwise feel free to contact me at `glen.cowan@cern.ch` for questions.

Exercise 1.1: Show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

(Express $A \cup B$ as the union of three disjoint sets.)

Exercise 1.2: A beam of particles consists of a fraction 10^{-4} electrons and the rest photons. The particles pass through a double-layered detector which gives signals in either zero, one or both layers. The probabilities of these outcomes for electrons (e) and photons (γ) are

$$P(0 \mid e) = 0.001 \quad \text{and} \quad P(0 \mid \gamma) = 0.99899$$
$$P(1 \mid e) = 0.01 \qquad\qquad\quad P(1 \mid \gamma) = 0.001$$
$$P(2 \mid e) = 0.989 \qquad\qquad\quad P(2 \mid \gamma) = 10^{-5}.$$

(a) What is the probability for a particle detected in one layer only to be a photon?

(b) What is the probability for a particle detected in both layers to be an electron?

Exercise 1.3: Consider a random variable x and constants α and β. Show that

$$\begin{aligned}
E[\alpha x + \beta] &= \alpha E[x] + \beta, \\
V[\alpha x + \beta] &= \alpha^2 V[x].
\end{aligned} \tag{1}$$

Exercise 1.4: Consider the exponential p.d.f.,

$$f(x; \xi) = \frac{1}{\xi} e^{-x/\xi}, \quad x \geq 0. \tag{2}$$

(a) Show that the corresponding cumulative distribution is given by

$$F(x) = 1 - e^{-x/\xi}, \quad x \geq 0. \tag{3}$$

(b) Show that the conditional probability to find a value x less than $x_0 + x'$ given that $x > x_0$ is equal to the (unconditional) probability to find x less than x', i.e.

$$P(x \leq x_0 + x' | x \geq x_0) = P(x \leq x'). \tag{4}$$

(c) Cosmic ray muons produced in the upper atmosphere enter a detector at sea level, and some of them come to rest in the detector and decay. The time difference t between entry into the detector and decay follows an exponential distribution, and the mean value of t is the mean lifetime of the muon (approximately 2.2 μS). Explain why the time that the muon lived prior to entering the detector does not play a role in determining the mean lifetime.