The semi-empirical mass formula (SEMF)

The binding energy of a nucleus can be modelled as

$$B(Z,A) = a_v\,A - a_s\,A^{2/3} - a_C\,Z(Z-1)A^{-1/3} - a_{\rm sym}\,(A-2Z)^2/A + \begin{cases} -a_p\,A^{-3/4} \\ 0 \\ +a_p\,A^{-3/4} \end{cases}$$

where, for best agreement with experiment, $a_v =$15.5 MeV, $a_s =$16.8 MeV, $a_C =$0.72 MeV, $a_{\rm sym} =$23 MeV, $a_p =$34 MeV.

The semi-empirical mass formula is then

$$\begin{aligned} M(Z,A) &= Zm(^1H) + Nm_{\rm n} - B(Z,A)/c^2 = \\ &= Zm(^1H) + (A-Z)m_{\rm n} - B(Z,A)/c^2 \end{aligned}$$

Isobar mass chains

For isobars (constant A), the SEMF is a parabola depending on Z:

$$M(Z,A) = C + DZ + EZ^2$$

with

$$\begin{aligned} C &= (-a_v\,A + a_s\,A^{2/3} + a_{\rm sym}\,A + \delta)/c^2 + A\,m_{\rm n} \\ D &= (-a_C\,A^{-1/3} - 4a_{\rm sym})/c^2 + (m(^1H) - m_{\rm n}) \\ E &= (+a_C\,A^{-1/3} + 4\frac{a_{\rm sym}}{A})/c^2 \end{aligned}$$

with minimum at:

$$\frac{\partial M}{\partial Z})_A = 0 \Leftrightarrow D + 2EZ = 0 \Leftrightarrow Z_{\rm min} = -\frac{D}{2E}$$

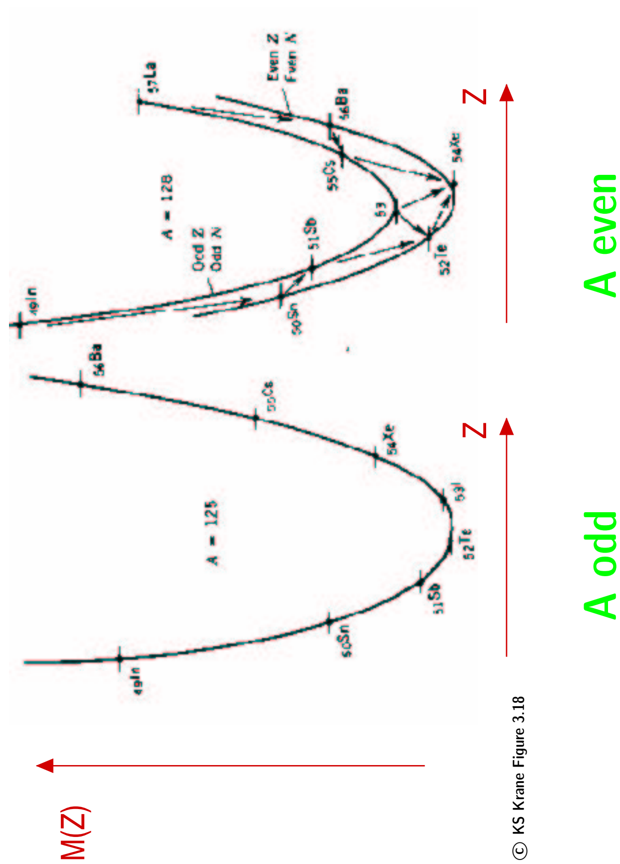
$$Z_{\rm min} = \frac{(m_{\rm n} - m(^1H))c^2 + a_C\,A^{-1/3} + 4a_{\rm sym}}{2a_C\,A^{-1/3} + 8a_{\rm sym}A^{-1}}$$

Isobar decay chains

If $M(Z_{\text{initial}}, A)$ is sufficiently larger than $M(Z_{\text{final}}, A)$ then the nuclear beta transition

$$M(Z_{\text{initial}}, A) \rightarrow M(Z_{\text{final}}, A)$$

is energetically favoured.
(β-decay promotes nuclear transitions between isobars, n$\leftrightarrow$p)



Isobar decay chains (cont'd)

Note:

- β^--decay increases Z, β^+-decay decreases Z

- the mass difference between neighbouring isotopes increases as one gets away from the “valley” of stability

For A even, note also that

- due to the nonzero pairing term Ô two parabolas displaced by 2δ;

- some nuclei can decay in either direction, β^- or β^+ (e.g., $^{128}_{53}I$)

- $^{128}_{52}Te$ cannot decay to $^{128}_{53}I$; however it is energetically possible for it to undergo double-beta decay ($\beta\beta$): $^{128}_{52}Te \rightarrow ^{128}_{54}Xe + 2e^-$.