

Magic numbers

Experimental evidence for **nuclear shell structure**:

+ **proton & neutron separation energies** show similar behaviour to atomic ionization energies (i Fig. 5.1 & 5.2)

Smooth increases correspond to filling of shell; sudden jumps indicate transitions to next shell.

+ **Neutron capture probability** drops drastically for nuclei with $N = \dots, 50, 82, 126$ (i Fig. 5.3b)

Magic numbers of nucleons:

nuclei with Z or $N = 2, 8, 20, 28, 50, 82, 126$ are extremely stable.

- Nuclei with Z and N magic numbers are said to be **doubly magic**
 Ü e.g., ^{78}Ni .

Nucleon separation energies

Compare **ionization energies** in atoms...

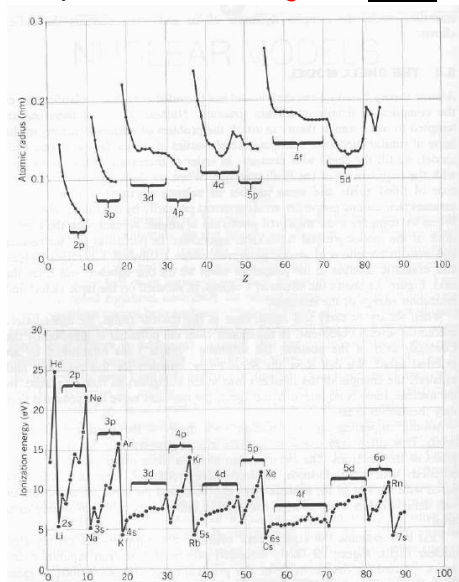
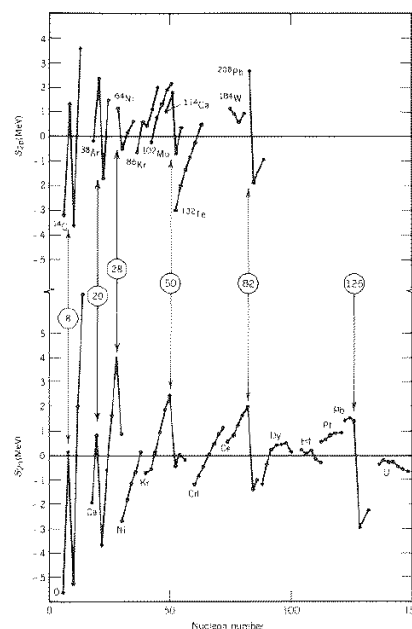


Figure 5.1 Atomic radius (top) and ionization energy (bottom) of the elements. The smooth variations in these properties correspond to the gradual filling of an atomic shell, and the sudden jumps show transitions to the next shell.

...with **n/p separation energies** in nucleii:



The nuclear shell model (I)

+ Try to account for the observed magic numbers:

- The **harmonic oscillator** potential and the **infinite square well** potential manage to reproduce **some** of these magic numbers... **but not all**:

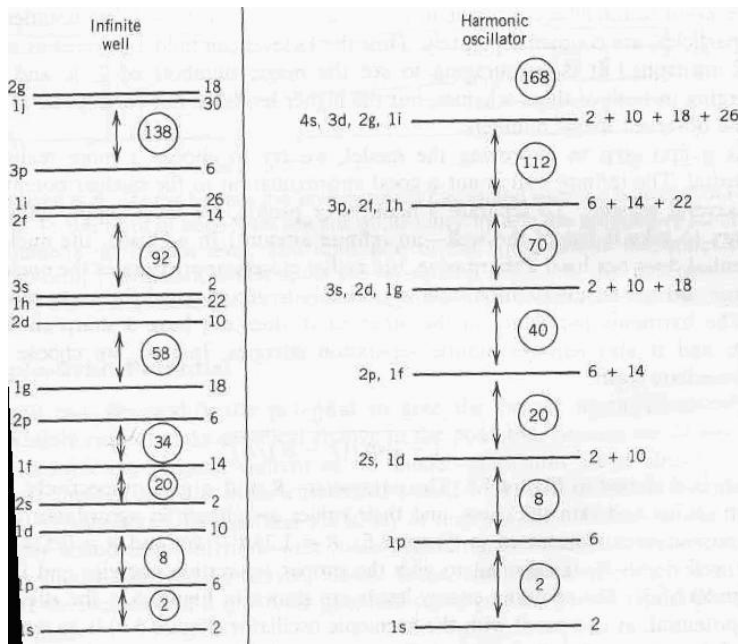
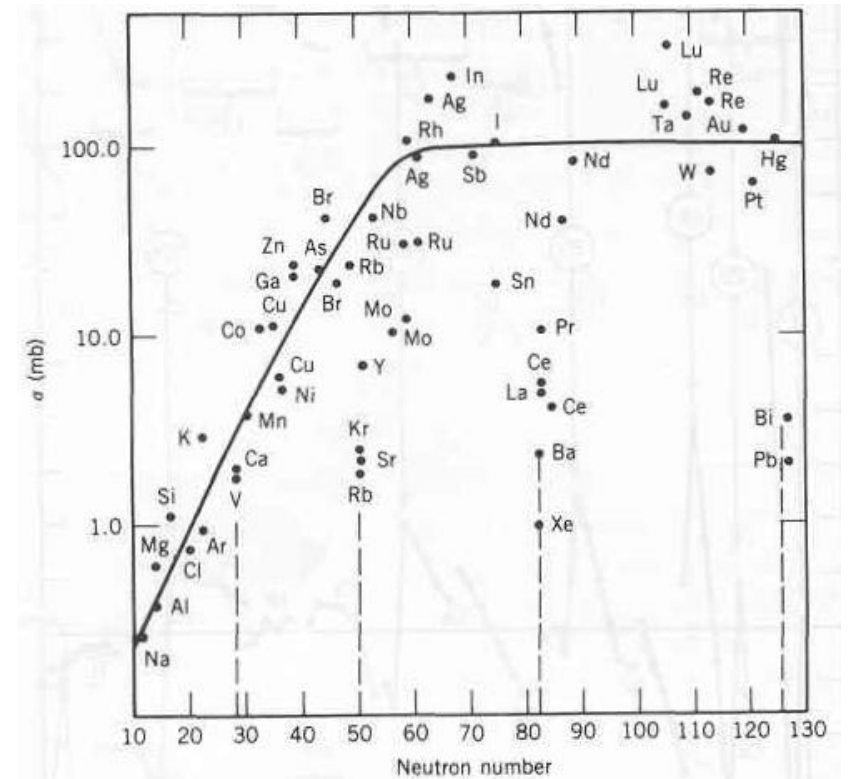


Figure 1: Shell structure obtained with infinite well and harmonic oscillator potentials. The capacity of each level is indicated to its right. Large gaps occur between levels, which we associate with closed shells. The circled numbers indicate the total number of nucleons at each shell closure. © KS Krane, Figure 5.4

Neutron capture probability



© KS Krane Figure 5.3

- + marked **drop** in neutron capture probability for nuclei with $N = 28, 50, 82, 126$

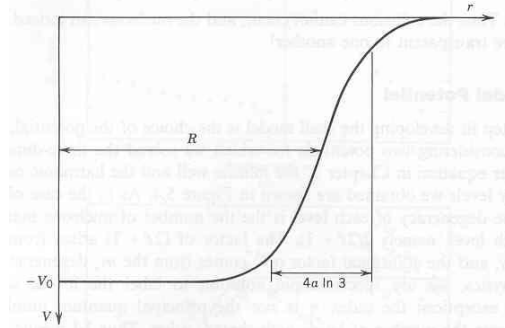
The nuclear shell model (II)

Improving the model \Rightarrow choice of potential:

Potential	Comments
infinite square well	<ul style="list-style-type: none"> 7 infinite nucleon separation energy 7 nuclear boundary too sharp
harmonic oscillator	<ul style="list-style-type: none"> 7 nuclear boundary not sharp enough

More realistic: shell-model potential:

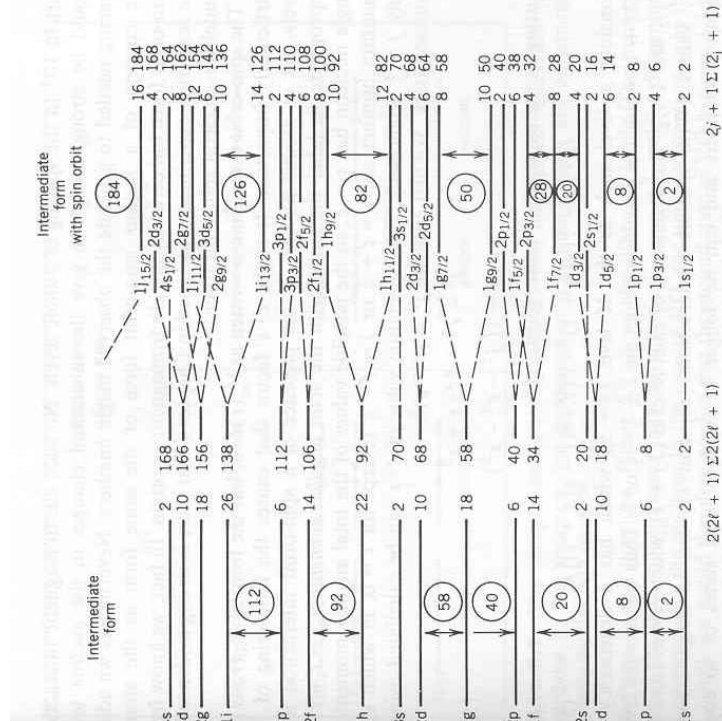
$$V(r) = \frac{-V_0}{1 + \exp[(r - R)/a]}$$



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The nuclear shell model (III)

- Using the shell-model potential and adding the nucleon spin-orbit interaction results in a distribution of the energy levels that matches exactly the observed magic numbers



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